

# The redshift in Hubble's constant

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## Abstract

A generalisation to electrodynamics and Yang-Mills theory is presented that permits computation of the speed of light. The model presented herewithin indicates that the speed of light is not a universal constant. This may be relevant to the current debate in astronomy over large values of the Hubble constant obtained by the latest generation of ground and space-based telescopes. An experiment is proposed based on Compton scattering.

## 1 Introduction

Astronomers have recently reported on observations for the Hubble constant that predict an age for the universe younger than the estimated age of some globular clusters [11][4][14]. The obvious tension contained in this result has been christened the 'age crisis'. There are three possibilities of course: the measurements made for the Hubble constant are incorrect; current models for

stellar evolution are incorrect; or, there is new physics to be understood. Without further observation it is too early to view favourably recent experiments for the Hubble constant. And as noted by Sandage, if the results are supported by further experimentation, it must be understood why previous estimates for the Hubble constant based on observations of type Ia supernovae are only half as large. It may be that models for stellar evolution need to be re-evaluated as well.

Over the course of the last fifteen years, observation has uncovered deviations in the Hubble constant. The Hubble relation turns sharply upwards from linearity on a redshift vs. distance plot [8][13]. Refinements to the distance scale achieved by [11][4][14] cannot of course account for the non-linearity in the Hubble constant. Sandage has argued that the non-linearity is due to a bias in the choice of objects chosen by astronomers for study, and that at large distances such a bias must be filtered out in order to correctly determine the Hubble constant. Such is the current debate.

Another hypothesis is suggested by the deviations in the Hubble constant: the speed of light in vacuum at space-time points remote from Earth is less than the present, terrestrial speed of light. For suppose that the speed of light in a vacuum were not an absolute constant, then the red-shift would need to be reassessed. The redshift is given by

$$\frac{\Delta\lambda}{\lambda} \cong \frac{v}{\tilde{c}}, \quad (1)$$

where we assume that the velocity of the emitter  $v \ll \tilde{c}$ , the speed of light when the photon is emitted. If the speed of light at the time of emission is smaller than the present speed of light on earth, then the observed red-shift in (1) would be greater. This would make the Hubble constant larger, and thereby make the universe appear to be younger than it actually is. A lower speed of light in the early universe might therefore be helpful in understanding the purported ‘age crisis’. In addition, one notes that under these circumstances the enormous powers associated with quasars would be reduced. In the next section we present

a model that permits us to derive the speed of light in a natural way. In section three we propose an experiment based on Compton scattering that might detect variations in the speed of light.

## 2 A model

Let  $\pi : P \rightarrow M$  be a principal  $U(2)$ -bundle over an oriented, compact, connected four-manifold  $M$ , and denote by  $E$  the associated rank two adjoint vector bundle.  $\mathcal{A}(P)$  is the space of connections on  $P$ . Let  $A, B \in \mathcal{A}(P)$ , and introduce local coordinate charts with indices  $\mu = 0, \dots, 3$  on  $M$ . The Lie algebra-valued connections or vector potentials,  $A_\mu$  and  $B_\mu$ , induce exterior covariant derivatives  $D_\mu^A = \partial_\mu + A_\mu$  and  $D_\mu^B = \partial_\mu + B_\mu$  on the associated adjoint vector bundle  $E$ . The curvatures  $H^A$  and  $K^B$  are defined by  $2D_{[\mu}^A D_{\nu]}^A s = H_{\mu\nu}^A s$  and  $2D_{[\mu}^B D_{\nu]}^B s = K_{\mu\nu}^B s$ . In this way  $H^A$  and  $K^B$  are two-forms on  $M$  taking values in  $E$ , that is  $H^A, K^B \in \Lambda^2(M, E)$ . The Lagrangian Action that forms the basis of our model is given by

$$\mathcal{L}(A, B) = \int_M \langle (H^A \otimes I_E) \wedge (I_E \otimes K^B) \rangle - \frac{1}{2} \langle (I_E \otimes K^B)^2 \rangle, \quad (2)$$

where  $H^A$  and  $K^B$  are gauge field curvatures over the four-manifold,  $M$ .  $I_E$  denotes the identity transformation on the adjoint bundle,  $E$ , and the bundle inner product is represented by  $\langle \rangle$ . The Killing-Cartan inner product can be adopted. The inner product is normalized so that  $\langle I_E^2 \rangle = 1$ . The form of the Lagrangian (2) generalises the topological gauge field theories studied some time ago by Horowitz [6]. In local space-time coordinates and using the Killing-Cartan inner product the Lagrangian Action can be written explicitly as

$$\begin{aligned} \mathcal{L}(A, B) = & \int_M H_{[\mu\nu]}^a K_{\lambda\rho]}^b \operatorname{tr}(T^a I_E) \operatorname{tr}(T^b I_E) d^4x \\ & - \frac{1}{2} \int_M K_{[\mu\nu]}^a K_{\lambda\rho]}^b \operatorname{tr}(I_E I_E) \operatorname{tr}(T^a T^b) d^4x. \end{aligned} \quad (3)$$

The generators of the Lie algebra are denoted by  $T^a$ . For gauge groups where  $\operatorname{tr}(T^a) = 0$ , the Lagrangian (3) reduces to the second integral—these are the

topological field theories studied by Baulieu and Singer [2]. The variational field equations for the Lagrangian (2) are

$$D^A K^B = 0, \quad D^B H^A = 0. \quad (4)$$

The field equations are clearly independent of any metric structure on  $M$ . The set of solutions to (4) is clearly not trivial, because when  $A = B$  the field equations reduce to the Bianchi identities.

Observe that when a space-time metric is placed on  $M$  (by whatever means) and used to define the Hodge star operator,  $*$ , the topological field equations (4) become the source-free Yang-Mills or electrodynamic field equations when the gauge field  $B$  is chosen so that  $K^B = *H^A$ . The topological field theory above therefore, is seen to contain Yang-Mills theory and electromagnetism. It is for this reason that we defined a field theory with *two* vector potentials,  $A$  and  $B$ , and not one.

We turn now to the Bogomol'nyi structure. By completing the square, the Lagrangian (2) can be rewritten as

$$\begin{aligned} 2\mathcal{L} = \int_M < (H^A \otimes I_E - I_E \otimes K^B)^2 > \\ - \int_M < (H^A \otimes I_E)^2 > . \end{aligned} \quad (5)$$

The Lagrangian (5) is now in Bogomol'nyi form. The Bogomol'nyi equations arising from (5) are

$$H^A \otimes I_E - I_E \otimes K^B = 0. \quad (6)$$

By an index computation equations (6) imply that  $H^A$  and  $K^B$  are projectively flat,

$$H^A = K^B = iFI, \quad (7)$$

where  $F$  is a real-valued two form on  $M$ . Clearly solutions to the Bogomol'nyi equations (6) automatically satisfy the variational field equations (4) when  $F$  is closed.

Let  $E_A$  and  $E_B$  be the adjoint vector bundles equipped with  $D^A$  or  $D^B$ , respectively.  $E^*$  is the dual bundle to  $E$ . The curvature of the tensor product

bundle  $E_A \otimes E_B^*$  is given by [7]

$$\Omega_{E_A \otimes E_B^*} = H^A \otimes I_E - I_E \otimes K^B.$$

In view of this, the Bogomol'nyi equations in (6) are seen to be a vanishing curvature condition on the tensor product bundle  $E_A \otimes E_B^*$ , and implies that

$$c_2(E \otimes E^*) - \frac{1}{2}c_1(E \otimes E^*)^2 = 4c_2(E) - c_1(E)^2 = 0. \quad (8)$$

The second integral in (5) can also be written as a characteristic class,

$$\int_M \text{tr}(H^A \wedge H^A) = 8\pi^2(c_2(E) - \frac{1}{2}c_1(E)^2) = -8\pi^2 ch_2(E). \quad (9)$$

The second Chern character is denoted by  $ch_2(E)$ . It is clear that under a perturbation of the vector potentials both integrals in (5) are invariant, and when the Bogomol'nyi equations are satisfied the Lagrangian is proportional to the characteristic class (9). Non-singular solutions to the Bogomol'nyi equations are non-trivial and stable when (9) is non-vanishing.

We have therefore identified a class of non-singular, finite-energy, stable solutions to the variational field equations (4). However, most solutions to the Bogomol'nyi equations do not appear to be 'particle-like'. Thus within the class of Bogomol'nyi solutions we must now define the 'particle-like' solutions to the field equations: solutions to the Bogomol'nyi equations (7) will be said to be 'particle-like' if together they form a phase space manifold that is non-singular, Hausdorff, and of finite dimension. This is a tacit assumption in the particle picture, both classically and quantum mechanically. The phase space is given by the moduli space of solutions to the Bogomol'nyi equations (7) defined over space-time,  $M$ . The phase space is generally of infinite dimension and is not necessarily Hausdorff. In algebraic geometry, mathematicians look for Mumford-Takemoto topological stability to ensure that moduli spaces are Hausdorff. Kobayashi has reformulated Mumford-Takemoto stability into a differential geometric form, known as the Einstein-Hermitian condition [7]. As we shall see, the Einstein-Hermitian condition is equivalent to restricting to those

solutions of the Bogomol'nyi equations that are compatible with some additional geometric structure.

To implement the Einstein-Hermitian condition we assume that space-time,  $M$ , is now a compact Kähler manifold of real dimension four with a Kähler form  $\Phi$ , and that  $E \rightarrow M$  is holomorphic. A Hermitian metric  $h$  and a holomorphic structure  $\bar{\partial}$  on  $E$  give rise to a unique connection,  $A$ . The associated curvature  $H^A$  is of type  $(1,1)$ , that is,  $H^A \in \Lambda^{1,1}(M, E)$ . The mean curvature,  $K$ , of the vector bundle over a Kähler manifold is given by [7, p.99]

$$K = i\Lambda H^A,$$

where the operator  $\Lambda : \Lambda^{n,m} \rightarrow \Lambda^{n-1,m-1}$  is defined as the adjoint operator to  $L \equiv \Phi \wedge \cdot$ . A vector bundle is said to be Einstein-Hermitian when

$$K = i\Lambda H^A = kI_E, \tag{10}$$

for  $k$  a real constant. When  $k$  is a non-constant function on  $M$ , then the vector bundle is said to be weak Einstein-Hermitian.

One cannot help but notice the similarity of equation (10) with the projective flatness

$$H^A = FI_E, \tag{11}$$

of the Bogomol'nyi equations (7). For holomorphic vector bundles  $F \in \Lambda^{1,1}(M)$ . Applying the operator  $\Lambda$  to both sides of (11), we see that a projectively flat holomorphic vector bundle over a Kähler surface is weak Einstein-Hermitian:

$$K = i\Lambda H^A = i(\Lambda F)I_E = \varphi(x)I_E,$$

with  $\varphi(x)$  a real-valued function on  $M$ . In addition, it can be shown that there exists a conformal change to the Hermitian structure  $h \rightarrow h' = \alpha h$  such that  $(E, h')$  satisfies the Einstein-Hermitian condition with a constant factor  $k$  [7, p.104].  $k$  depends only on  $c_1(E)$  and the cohomology class of  $\Phi$ , in particular it is independent of the Hermitian inner product  $h$ . Now in the other direction,

by a theorem of Lübke it is known that all Einstein-Hermitian vector bundles satisfying the topological requirement (8) are holomorphic projectively flat [9]. Thus, up to a conformal change in the Hermitian structure, a holomorphic projectively flat connection and an Einstein-Hermitian connection are equivalent.

To obtain a phase space that is topologically well-behaved we shall therefore restrict to the set of holomorphic projectively flat connections on a vector bundle  $E$  satisfying the topological condition (8), equivalently, restrict to Einstein-Hermitian connections. We shall call the Einstein-Hermitian Bogomol'nyi solitons 'topological instantons', because of the obvious similarity between these objects and the self-dual instantons in Yang-Mills theory. Topological instantons are naturally parametrized by a continuous parameter, the Einstein-Hermitian constant  $k$  in (10). For fixed  $k$ , the phase space is denoted by  $\mathcal{M}_k$ .

Now let the underlying space-time manifold be a flat Kähler complex two-torus (real dimension four); we assume a space-time that is periodic in both space and time. All complex tori admit a Kähler structure: a Kahler metric  $g$  and a closed Kahler form  $\Phi \in \Lambda^{1,1}(M)$  [15]. The complex rank two vector bundle  $(E, h) \rightarrow (M, g, \Phi)$  defined over the Kähler torus is assumed to have the Chern numbers  $4c_2(E) = c_1(E)^2 = -4$ .  $h$  is the Hermitian metric on  $E$  which can be constructed using the Killing-Cartan form. The significance behind the choice of topology is that such a bundle satisfies the topological condition (8) necessary for the bundle to admit projectively flat connections, and the Lagrangian (9) equals  $-8\pi^2$  so that *stable* topological instantons might exist. For abelian varieties existence theorems are known [10]. We shall study 'diagonal'  $U(2)$  topological instantons on this bundle. By 'diagonal' we mean that the Einstein-Hermitian connections  $A$  and  $B$  are equal ( $A = B$ ). Diagonal instantons are examined because there is little physical evidence to suggest two distinct vector potentials. We take the constant  $k$  in the Einstein-Hermitian condition (10) to be fixed and non-zero. The Kähler structure on  $M$  allows us to study the phase space,  $\mathcal{M}_k$ .

As we have seen the phase space,  $\mathcal{M}_k$ , of  $U(2)$  topological instantons on the holomorphic Hermitian vector bundle  $(E, h) \rightarrow M$  is equivalent to the moduli space of irreducible Einstein-Hermitian connections  $\mathcal{E}(E, h)/U(E, h)$  on  $(E, h)$ .  $U(E, h)$  denotes the unitary gauge transformations of  $(E, h)$ . The complex dimension of the  $U(2)$  topological instanton phase space when non-empty is given by [7]:

$$\dim_{\mathbf{C}}(\mathcal{M}_k) = 4h^{0,1}(M) - 6. \quad (12)$$

The Kähler torus has  $h^{0,1}(M) = 2$ . Therefore if a (diagonal) topological instanton exists, the real dimension of the phase space,  $\mathcal{M}_k$ , is four. Note that massless particles have phase spaces of real dimension four in (3+1) space-time. (We have skipped over K3 surfaces as models for space-time because they have  $h^{0,1}(M) = 0$ , thereby giving phase space a negative dimension.)

The next order of business is to include special relativity into the theory locally by modelling  $\mathcal{M}_k$  with  $\mathcal{M}_{massless}$ , the phase space for massless particles. Consider the phase space for massless particles in  $\mathbf{R}^3$ . The (covariant) phase space is equivalent to the space-of-motions. Massless particles in  $\mathbf{R}^3$  move on straight lines and at the speed of light,  $c$ . We may therefore parameterize the possible motions of a massless particle by assigning to a straight line,  $\mathbf{x}(t)$ , in  $\mathbf{R}^3$  a velocity vector  $\mathbf{c} = c\hat{\mathbf{n}}$ , and a position vector  $\mathbf{d}$  so that  $\mathbf{x}(t) = \mathbf{d} + t\mathbf{c}$ . The position vector  $\mathbf{d}$  is defined as the normal from the origin to the line,  $\mathbf{x}(t)$ , equivalently, the point on the line nearest the origin. Thus the phase space is

$$\mathcal{M}_{massless} \cong \{(\mathbf{c}, \mathbf{d}) \in S_c^2 \times R^3 \mid \mathbf{c} \cdot \mathbf{d} = 0\}. \quad (13)$$

The phase space of the massless particle on  $\mathbf{R}^3$  is equivalent to the tangent bundle  $TS_c^2$  where the radius of the sphere is the speed of light. The natural metric on  $TS_c^2$  is given by

$$ds^2 = f(r)dr^2 + a(r)(d\psi + \cos\theta d\varphi)^2 + c^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (14)$$

where  $(\theta, \varphi)$  are spherical polar coordinates on the two-sphere, and  $(r = \|\mathbf{d}\|, \psi)$  plane polar coordinates on the tangent plane. The functions  $f(r)$  and  $a(r)$  are



arbitrary, and we have multiplied  $c$  by unit time. We require that the local geometry of  $\mathcal{M}_k$  reduce to the geometry of special relativity.

For a Kähler-torus space-time the phase space manifold of topological instantons,  $\mathcal{M}_k$ , was shown by Kobayashi to be symplectic Kähler [7]. This means that the holonomy group of  $\mathcal{M}_k$  is  $Sp(1) \simeq SU(2)$ , and immediately implies that the Ricci tensor vanishes. Since  $\mathcal{M}_k$  is of real dimension four, a Ricci-flat Kähler manifold is in fact hyper-Kähler. Hyper-Kähler metrics on four-manifolds have been studied in depth since they have been found to be important to both the gravitational instanton and the BPS magnetic monopole [5][1]. Our goal is now to determine the possible hyper-Kähler metrics on  $\mathcal{M}_k$ .

Assume that the hyperKähler metric on  $\mathcal{M}_k$  is complete and non-singular on an open set of the topological instanton phase space,  $U \subset \mathcal{M}_k$ . Assume local isotropy of the universe, so that the phase space  $\mathcal{M}_k$  admits  $SO(3)$  as a group of local isometries. Let us also assume that on  $U$  the orbits defined by the action under the isometries are three-dimensional. Then, the only complete, non-singular,  $SO(3)$ -invariant hyperKähler metrics on four-manifolds with three-dimensional orbits are: the flat metric, the Atiyah-Hitchin metric, the Taub-NUT metric with positive mass, and the Eguchi-Hanson metric [5]. The Taub-NUT and Eguchi-Hanson metric admit another  $U(1)$  so they are in fact  $U(2)$ -invariant. Only Taub-NUT and the Eguchi-Hanson metric appear to be compatible with the massless particle metric (14). If we also place on  $M_{massless}$  its natural complex structure and note that it is invariant under the natural  $SO(3)$  action, then only the Eguchi-Hanson metric is compatible with the local geometry of special relativity. The Eguchi-Hanson metric is of the form

$$ds^2 = \left[ 1 - \left( \frac{a}{r} \right)^4 \right]^{-1} dr^2 + \frac{r^2}{4} \left( \sigma_1^2 + \sigma_2^2 + \left( 1 - \left( \frac{a}{r} \right)^4 \right) \sigma_3^2 \right),$$

where  $\{\sigma_i\}$  is the dual basis for  $so(3)$ . In terms of Euler angles, we define

$$\begin{aligned}\sigma_1 &= d\varphi \sin \theta \cos \psi - d\theta \sin \psi, \\ \sigma_2 &= d\varphi \sin \theta \sin \psi + d\theta \cos \psi, \\ \sigma_3 &= d\varphi \cos \theta + d\psi.\end{aligned}$$

In Euler coordinates the Eguchi-Hanson metric becomes

$$ds^2 = [\gamma(r)]^{-1} dr^2 + \frac{r^2}{4} \gamma(r) (d\psi + \cos \theta d\varphi)^2 + \frac{r^2}{4} (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (15)$$

where  $\gamma(r) \equiv 1 - (a/r)^4$ . Compare (15) with (14). Note that while the metrics are similar, the two-sphere in the Eguchi-Hanson metric is a function of  $r$ . With this observation our physical parameterization of the open set follows.

The Euler angles  $(\theta, \varphi, \psi)$  in the Eguchi-Hanson metric (15) define the direction of the propagation, and the radius of the sphere is the speed of the massless topological instanton, as we saw for the massless particle above. For the instanton to carry energy and momentum and still remain massless, the energy-momentum relation in special relativity implies that the topological instanton should move at the speed of light. Thus the speed of light in this theory is also subject to spatial variation. However, the spatial coordinate  $r$  in the tangent space is a little ambiguous, e.g., where is the origin? We shall suppose that the origin is Earth, and that  $r = 2c - \alpha \parallel \mathbf{d} \parallel$  where  $c$  is the present terrestrial speed of light in vacuum multiplied by unit time,  $\alpha$  is a dimensionless constant, and  $\parallel \mathbf{d} \parallel$  is the distance from Earth. This is our physical parameterization of  $\mathcal{M}_k$ .

The speed of light is an experimental constant on length scales much smaller than cosmological scales, so that we may continue to use the energy-momentum relation locally. Any variation in the speed of light in vacuum is presumably on very large spatial scales.

### 3 An experiment is proposed

Let us propose then how one might detect a difference in the speed of distant light when compared with terrestrial light,  $c$ .

Assume that the speed of light  $\tilde{c}$  is not an absolute constant when viewed at very large spatial scales. To measure deviations in speed between distant light,  $\tilde{c}$ , and terrestrial light,  $c$ , one presumably examines photons that have interacted with matter in the early universe and have, since then, traveled unimpeded through space. Some of these photons eventually enter a detector (unimpeded travel may require that the detector be space-based). Since it is assumed that no interaction occurs during the photons' long journey, the energy,  $E$ , and the linear momentum,  $E/\tilde{c}$ , are conserved. This implies that the photons travel toward Earth with the constant speed of light,  $\tilde{c}$ , given to them upon emission. The incoming photons are absorbed by the matter in the detector, and are re-emitted (photon scattering). We shall assume that the energy and linear momentum are conserved in photon scattering. By studying the scattered photons we determine characteristics of the incoming photons. This is the Compton effect, of course. A derivation is now given for the lowest-order correction to the Compton formula when the speed of the incident photon is not the terrestrial speed of light,  $c$ . For simplicity, we assume that the photon scatters off a loosely bound electron in the detector (a graphite detector, for example).

Let the incident photon have energy  $E$  and momentum  $E/\tilde{c}$ , where  $\tilde{c}$  is the incident speed of light. The electron with mass  $m$  is assumed to be initially at rest. After interacting with the electron, the scattered photon has energy  $E'$  and momentum  $E'/c$ , and the electron has momentum  $\mathbf{P}'$ . Let  $c = \tilde{c} + \Delta c$ , define  $\epsilon \equiv \Delta c/\tilde{c}$ , and use conservation of momentum and conservation of energy to obtain

$$\epsilon^2 E^2 + 2\epsilon E(E - E' \cos \theta) + 2EE'(1 - \cos \theta) = 2mc^2(E - E'). \quad (16)$$

We divide (16) by  $EE'$  and use  $E = h\tilde{c}/\lambda$ ,  $E' = hc/\lambda'$  to give

$$\frac{\epsilon^2}{1+\epsilon} \frac{\lambda'}{\lambda} + 2\epsilon \left( \frac{1}{1+\epsilon} \frac{\lambda'}{\lambda} - \cos\theta \right) + 2(1 - \cos\theta) = \frac{2mc}{h}(\lambda' - \lambda(1+\epsilon)),$$

where we have used

$$\frac{\tilde{c}}{c} = \frac{1}{1+\epsilon}. \quad (17)$$

Define  $\Delta\lambda = \lambda' - \lambda$  and take the lowest-order correction to the Compton formula to obtain

$$\Delta\lambda \cong \frac{h}{mc}(1 - \cos\theta) + \epsilon \left[ \lambda + \frac{1}{\lambda} \frac{h^2}{m^2 c^2} (1 - \cos\theta) \right].$$

The second term is dependent on the wavelength, while the terrestrial Compton effect is obviously independent of the wavelength. Scattering dependence on the incident wavelength would be a clear signal for spatial variation in the speed of light.

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